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ABSTRACT

This study assesses the abilities of kindergarten children who attended the Mt. Druitt Early Childhood Project (MDECP) in Australia to apply logical processes to mathematic problems not generally found in the classroom. The mathematical abilities of 56 children enrolled in one of the MDECP's five programs (competency, cognitive, behaviorist, contemporary, and home-base) were compared with the mathematical abilities of 33 non- preschool experience children on seven different tasks (representation and memory, matrix, number operations, number patterns, conservation of number, measurement, and symmetry). In general, results indicated fairly consistent trends in favor of the preschool children, but most of these differences were not statistically significant. The only significant differences between the two groups were found in responses to problems in number operations and number patterns tasks. The most significant difference was found in the responses to the measurement task. The two programs which contributed to most of the variance were the behaviorist and contemporary programs. Significant sex differences related to the matrix reconstruction tasks were found (girls made more error than boys). Implications for the teaching and assessment of the children's mathematical abilities are discussed. (Author/MP)

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PROJECT EVALUATION REPORT

Mathematical Thinking in Kindergarten

Muriel Healey

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This draft paper reports specific aspects of evaluation activities not presented in detail in the Project's Final Report. Comments or requests for further information may be addressed to the Project, care of the above address.

Preface

This Kindergarten study continues on the evaluation of children's mathematical abilities based on clinical assessment begun in the Preschool classes. The original tasks were refined and new situations introduced to evaluate the higher cognitive development of the children in the study. Implications for the teaching and assessment of children's mathematical abilities are drawn at the conclusion of the study.

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Mt Druitt Early Childhood Project

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Introduction

At the end of the second year of data collection in the Mt Druitt Early Childhood Project a set of mathematical games was chosen to test the logical thinking of the experimental and control groups now in Kindergarten. In the selection of domains for sampling the strategies of our subjects, attention was given to spatial organisation, number operations, number patterns, logical inference, measurement and ordering. In addition, a problem was chosen which would shed light on the modes of representation of our subjects and the effects of memory upon the organisation of representation.

In the preschool year none of the tasks had provided information about the salience of perceptual and organisational features of stimulus materials, either because the verbal instruction or the necessity for verbal response had clouded the issue. In general we were guided by the difficulties and discoveries of the mathematics evaluation during the preschool year (Healey, 1978b), but we attempted to provide problems of greater complexity which would require more sophisticated strategies for their solution.

The aim of this mathematical assessment was to explore the abilities of children in the Project in applying logical processes to problems not generally found in the classroom but which might tap the ability of the child to generalise from such classroom experiences. The only formal skill which would be directly related to success in the tasks was counting. It was not essential in their solution in most cases but could provide a valuable check for actions or a strategy to assist in decision making.

Subjects

The five original preschool programs in the Mt Druitt Early Childhood Project were represented by 53 children and there were 33 non-preschool children. The total sample consisted of 17 children from the Competency program, 19 from the Cognitive program, 16 from the Behaviourist, 16 from the Contemporary Australian and 18 from the children who had been enrolled in Home-Base during their preschool year. All of these children were members of an experimental class except for the Home-Base group since only a portion of the original sample of mothers and fathers had retained their close association with the school. The group was not identical with the one tested in the preschool year (Healey, 1978b) since some children were substitutes for those who had left the district and also because of the addition of the non-preschoolers.

The data were collected during the third term of 1978. Three graduate testers spent relatively equal time at each school site. Two of the testers administered the matrix and the representation task, the third collected the data for the flotation task. The remaining tasks were shared. Two of the testers had helped pilot some procedures and develop test instructions. The writer supervised the administration and dealt with any queries about coding or interpretation of responses.

The Method

There were seven tasks which will be described in the order of their presentation to children. Each task was derived from a different source and several sub-tasks were devised following the pilot study of Kindergarten children not included in the experimental or control groups. The piloting of each instrument helped in providing information about the difficulty level for this age group. The seven tasks were:

Representation and Memory;
Matrix;
Number Operations;
Number Patterns;
Conservation of Number;
Measurement;
Symmetry.

Task 1

Representation and Memory

The task was divided into two parts:
a) immediate recall;
b) delayed recall following interference.

The original study was reported briefly by Sinclair (1970). The stimulus was a card (16cm x 21cm) on which was an arrangement of small wooden shapes about 3mm thick. Four configurations of wooden shapes were glued into position in the four quadrants of the card. The shapes were painted green, yellow, blue, red and white and the card was yellow. Two examples of the same configurations were available but each tester used the same one consistently. On one card the 5 square pattern (see Figure 1) was made of yellow and blue shapes (4 blue and 1 yellow) and on the other there were yellow and red shapes; the 3 square pattern was made up of blue shapes in one case, of red in the other. Since two different testers were using the materials this meant that one tester used a different colour set to the other, but these differences were distributed evenly at the different school sites.

a) Immediate recall: The tester put the card in front of the child for approximately 10 seconds, saying "Look at this pattern because I want you to be able to remember it". Then the tester gave the child a blank card of equal dimensions to the original and said "Can you make the pattern for me? Here are some shapes for you to use". In a small box were the necessary pieces to make the pattern and three extra pieces. While the child was reconstructing the pattern the original card was turned over and the tester made no comment. However, she recorded the response by drawing the shapes used by the child on a blank sheet in the positions chosen and with coded colour names (see Appendix A).

) Delayed recall after interference: Task 2 was then administered and this served to interfere with children's recall of the above representation. After doing Task 2 the tester asked the child "Do you remember the pattern I showed you before? Do you think you could make it again?" While speaking she showed the child the blank card and the shapes which had been returned to the box. There was no opportunity to see the original model.

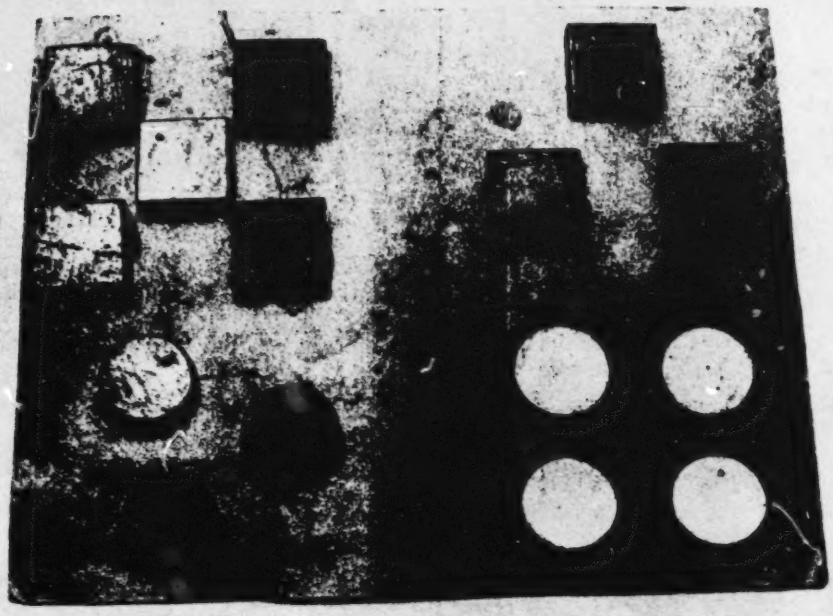


FIGURE 1: Shapes Used for Representation and Memory Task

Data Analysis

A careful analysis of each child's responses was performed by allocating quantitative ratings according to the various criteria of correct representation. The tester's individual records were the raw data for this analysis. The criteria were:

Correct Scores

Global representation:
This was derived from the general layout, whether the pattern was grouped in the four quadrants, and whether there was evidence of recall of the total configuration.

Configurations in each quadrant, correct on each of the following criteria:
rating 0-4
number, disregarding shape and colour: rating 0-4
colour, disregarding shape, number: rating 0-4
shape, disregarding number, colour: rating 0-4
position of the elements in each quadrant: rating 0-4.

Errors

Number of errors in each representation were grouped according to the following:
reversal of global pattern in terms of number, colour, shape and/ or position,
single block displacements,
additions of extra blocks to any configuration,

omissions of single blocks,
displacement of correct configurations (eg, into another quadrant),
incorrect orientation of configurations,
deviant pattern within configurations.

The same analytical processes were used with the delayed responses following the administration of Task 2.

Table 1 shows the numbers of children who were rated at each of the levels on each criterion of correct representation on immediate recall and after interference. Table 2 presents the mean scores for Task 1 and Table 3 lists the mean error scores for immediate and delayed recall.

TABLE 1 Numbers of children who correctly represented original pattern on immediate and delayed recall (N = 80)

Score	Global Score		Spatial Number		Colour		Shape		Block Position	
	I*	D*	I	D	I	D	I	D	I	D
0	6	6	10	10	9	10	7	5	12	11
1	5	3	12	10	4	6	3	3	15	19
2	10	9	19	16	20	16	11	11	17	16
3	17	12	27	27	20	20	23	21	11	8
4	42	50	12	17	27	28	36	40	25	26
Totals	80	80	80	80	80	80	80	80	80	80

* I = immediate recall D = delayed recall

TABLE 2 Representation and memory: Comparison of mean correct scores im immediate and delayed recall (N = 80)

Criterion	Mean Score		S.D.	
	Immediate	Delayed	Immediate	Delayed
Global	3.05	3.20	1.26	1.24
Number	2.24	2.39	1.26	1.29
Colour	2.65	2.63	1.30	1.36
Shape	2.98	3.10	1.24	1.17
Position	2.28	2.24	1.46	1.47

TABLE 3 Comparison of mean error scores immediate and delayed recall (N = 80)

Error type	Mean		S.D.		Error trend after delay
	Immediate	Delayed	Immediate	Delayed	
Reversal of global pattern features	.19	.20	.39	.40	→
Colour reversals	.33	.36	.70	.75	→
Shape reversals	.30	.29	.67	.62	←
Single block (Displacements)	.06	.06	.24	.24	=
Additions	.84	.70	.95	.78	←
Omissions	1.25	.95	2.20	1.96	←
Displaced configuration	.24	.20	.60	.50	←
Orientation	.59	.60	.59	.65	→
Deviant Pattern	1.60	1.65	1.3	1.4	→

Key to trend:

Increase in error →

Decrease in error ←

Equivalent degree of error =

Discussion of results

a) When accuracy of recall is taken as evidence of the salience of certain physical attributes it seems that shape is the best remembered feature (Table 1). Accuracy of recall increased with delay in the case of shape and also number, but not for colour or for block position (Table 1). Overall there was an improvement in general organisation following delay as shown in the overall or global score which reflects the grouping of elements into four configurations.

Error patterns follow the same trend in the case of shape and number. It seems that colour reversals and variations in spatial relations of the four configurations are increasingly responsible for error after interference but there is a decrease in number errors and shape reversals (Table 3).

It may be possible to attribute the relatively accurate memory for shape to the fact that only two shapes were used and their memory was reinforced by the shape of the configurations in which they served as structural components. It seems that number is also perceived by our subjects as a structural principle. The increased influence of these variables over time and across interference is compatible with the view that children who have attained logical structures are capable of reorganising memory and refining it (Furth, Ross & Youniss, 1974). Colour does not appear to be a salient variable in this process with these materials. Instead the variety of colour may interfere with accurate recall over the long term although it is readily recalled in the short term. Some of the children seemed to recall the contrast of the colours in a configuration rather than the absolute colour values. For instance, a colour reversal occurred when a child placed a red square in the centre of four yellows rather than the reverse. Colour contrasts may thus assist in the memory of configurations rather than the colours themselves.

b) When correct scores were analysed by multivariate analysis of variance in order to assess the effect of different programs with programs, sex and preschool or non-preschool as factors, no significant differences were found in either the immediate or delayed recall of pattern features. However, sex differences approached significance levels in the memory condition ($p<.085$); the univariate analysis suggested that the crucial variable in this difference was memory for shape and that the girls were superior in their results ($p<.015$).

Reanalysis on the basis of sex and preschool versus non-preschool experience yielded no significant differences in either the immediate or delayed reconstruction responses other than the sex differences already mentioned. The table of means shows a consistent trend towards higher scores in both representation and memory for the preschoolers and these differences were significant in the univariate tests. However, the error patterns were not consistent across samples. Non-preschoolers made less configuration errors of the types 'orientation' and 'displacement of correct configuration' in the immediate reconstruction, but did not hold this advantage after interference.

Task 2

The Matrix

In the preschool year of the Mt Druitt Early Childhood Project it had been found that the conceptual area which was most influenced by program effects was seriation and ordination (Healey, 1978b). The matrix problem (Bruner & Kenney, 1966) involves not only the ability to classify objects on two dimensions but to multiply their ordering. This is because each of the classes in the matrix also varies systematically on the other dimension. Each element must be considered as a member of both series and classes (Figure 2).

Also embodied in the matrix is the set of spatial relations among its elements (Steiner, 1974). The child was required to keep his memory of classes, series and spatial relations intact following rotation of the pattern. Moreover he is required to reconstruct the pattern after each rotation given only the position of one of the elements. A response which was mathematically correct (i.e. ordered correctly within series) might not be correct spatially unless the child could transpose the direction of each series.

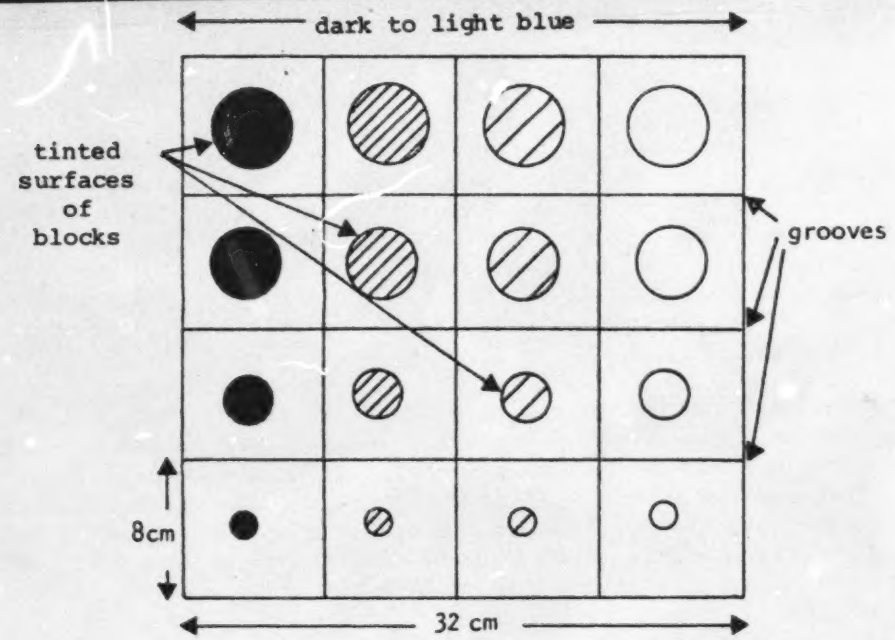
Materials and Procedures

A wooden board (32cm square) was grooved with coordinates to provide spaces for 16 circular blocks of uniform height (2 cm) which formed a matrix. The blocks varied in diameter (2 cm, 3.5 cm, 5 cm, 6.5 cm) and were painted in various tints of blue from very pale to very bright (by diluting a standard enamel with white paint). There were thus four classes of blocks according to diameter and four of tint. The standard arrangement was to place the large blocks in the row furthest from the child and to arrange them in series according to diameter so that the smallest were most proximal to the child. Also the blocks were arranged with the palest tint on the right border and the darkest on the left (Figure 2).

The child was first handed the blocks without the board and asked to sort them in piles. His spontaneous sorting preference, if any, was recorded. He was encouraged to make exhaustive groups according to the chosen dimension and then to choose another criterion for sorting. In each case he was asked to describe the classes of blocks and then, as the tester arranged them on the board, to describe each column and row. The words he used to describe the two series were also recorded. After he had studied the pattern the tester removed 2, 3 or 4 blocks in three practice trials. In each case 2, 3 or 4 spaces were left empty and the subject was asked to replace the blocks in them (Appendix B, Figure 1). Once he had done this correctly the tester removed all of the blocks from the board.

a) Reproduction

The tester said "Can you make the pattern we had before? Put all of the blocks on and make the pattern". Responses were recorded on a prepared grid designed by Steiner (1975) (Appendix B, Figure 2), so that the order of the replacement moves could be studied and the number of correct placements read off. The design of the grid and the number and coding of the blocks permitted some analysis of the salience of the dimensions, and the preference for use of individual blocks as 'land marks'.



Wooden board 32cm sq, polished

Block dimensions:

large	diam.	6.5cm
big	diam.	5cm
small	diam.	3.5cm
tiny	diam.	2cm

FIGURE 2 The Matrix

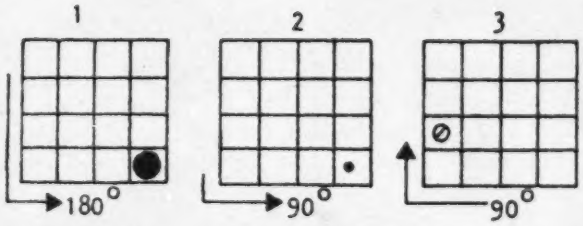


FIGURE 3 Transposition problems in the matrix

b) Transposition

Transposition I required the child to reconstruct the pattern after the tester had removed all but the largest bright blue block and, while the child watched, rotated the whole board 180°. The child was told to "make the pattern like it was before but with this block here. Think of all the blocks going for a ride just like this one did. Where would they end up? The pattern would be the same but it would be turned around like the board". Once again each move was recorded on a grid. At the end of the reconstruction the child was asked "Can we turn the board back like it was before? Will the pattern be the same?"

Transpositions II and III required similar instructions except that in each case the block remaining on the board was a different one and the rotation only 90°. The reproduction or partial reproduction of the patterns was coded on a grid.

If the child's reproduction was incorrect the tester replaced the blocks correctly before proceeding after turning the board back to its original position. If correct it was simply returned to the standard position before the blocks were removed for the next subtask.

Data analysis

a) Numbers of blocks correctly replaced were computed with the assistance of a template for each problem.
b) Multivariate Analysis of Variance was the method chosen for examining the effects of Program, Preschool experience and Sex on performance

Results

a) Frequencies of correct block replacements

Table 4 shows the frequencies of correct block replacements in each of the subtasks. More than half of our subjects attained full score in the reproduction of the standard. Slightly fewer were able to complete the 180° transposition correctly but the variance is greater with one quarter of the sample achieving only 4 or less correct block replacements. Nearly three quarters of the sample failed to achieve a score of 4 in the second transposition subtask because they failed to adjust the position of the axes represented by each class or series and thus made either the 'logical' but non-spatial response 'mirror image error', or became confused. In the third transposition subtask more subjects achieved a successful result once they had mastered the first few block placements, but there were some who were completely disoriented possibly due to the low salience of the key block. In this subtask it was impossible to make the mirror image error because of the asymmetrical position of the key block. After the initial dissonance about one third of the subjects found the solution and completed the subtask correctly.

b) Group differences

Multivariate analysis of variance of the correct block scores showed that there were significant differences ($p<.002$) between the sexes but no significant program differences were found. The univariate statistic indicated that the major differences in performance were in the first transposition subtask at 180° ($p<.006$). However, differences approached significance levels on each subtask. The mean scores in Table 5 reflect the

TABLE 4 Frequency of numbers of blocks correctly replaced in each of the four matrix tasks by individual children (N = 80)

N blocks placed correctly	Frequency of correct block replacements			
	Reproduction	Trans I	Trans II	Trans III
0	0	4	12	15
1	0	76	68	65
2	80	69	57	55
3	79	68	52	47
4	77	62	21	41
5	73	60	21	41
6	70	59	21	40
7	66	57	19	37
8	64	48	14	33
9	53	46	11	31
10	52	45	10	30
11	49	45	10	30
12	49	44	10	29
13	47	41	10	29
14	46	39	10	29
15	44	38	9	27
16	44	6	1	5

trends program by program. In four out of the five programs girls performed at a higher mean rate than boys in the first transposition task. The difference is in the reverse direction in the second transposition subtask at 90°, perhaps justifying the view that boys perform better spatially than girls. The difficulty in this task was primarily a spatial one, the problem of orientation, since the axes of the standard configuration reversed their horizontal/vertical relations.

There were no significant differences between the scores of preschoolers and non-preschoolers overall. However, Table 6 shows that preschoolers learned the pattern more quickly as shown in their reproduction scores and that the preschool boys were somewhat less perturbed by the axial change in Transposition II than other groups. Although the sex differences were more pervasive than the differences between children on the basis of experience in or out of preschool, the preschool girls were slightly better than their non-preschool peers.

Discussion of Results

a) Difficulty of transposition problems

The difficulty of the various subtasks is reflected in the mean scores which are recorded in Table 5:

Problem 1: With one exception (girls in the Home-Based program), scores decreased in Transposition I compared with reproduction. They were still relatively high and this may be due either to the salience of the key block as a 'marker' or to the fact that a 180° rotation retained the configuration relating to the axes but simply reversed it. It seems likely that both factors assisted the child in reconstructing the pattern. The children who coped well with the rotation and increased their scores were still learning the order of each series and their relations.

Problem 2 is much more difficult. The small block may be a less effective marker than the large one and it seems likely also that the directional change created an asymmetry of pattern which failed to be recognised as the same. The girls were affected by the change in configuration more than the boys.

Problem 3 is another 90° transformation, but because it provided information about the direction of the series in its key block the child who had mastered the multiplication of series got it right providing he was able to discriminate the relative colour value of the block. Many of the boys did not discriminate the colour well enough to make the correct decision about block positions in the colour series even if they were correct about positions in the size series. Often the two middle colour values were confused. An initial confusion could lead to a loss of the total configuration. Generally speaking the boys' scores were not affected as much as the girls' scores were by the 90° rotation but many did not improve their scores in the final task. This suggests that some of them had not mastered the seriations. Such sex differences in seriation were not apparent in our preschool sample (Healey, 1978b, p.25) and may have been an artifact of the materials (i.e. colour tone).

b) Strategies

The choice of the initial block by each child was an indicator of the salience of the blocks for pattern reconstruction. In each subtask, block 1 (the largest bright blue block) was selected by some of the children.

TABLE 5 Means and standard deviations, Program x Sex (N = 86)

Program	N Subjects	Reproduction Mean	Reproduction SD	Transposition I Mean	Transposition I SD	Transposition II Mean	Transposition II SD	Transposition III Mean	Transposition III SD
Cognitive									
Boys	9	10.56	(5.48)	7.56	(5.81)	3.44	(4.48)	9.11	(5.86)
Girls	10	14.80	(3.80)	11.90	(3.54)	2.30	(1.25)	7.90	(6.54)
Competency									
Boys	8	12.88	(4.90)	6.25	(5.97)	5.38	(6.32)	5.38	(6.21)
Girls	9	12.89	(3.76)	11.89	(5.67)	3.89	(4.76)	8.00	(6.98)
Contemporary									
Boys	8	11.25	(5.23)	8.75	(6.43)	6.25	(5.97)	5.00	(4.99)
Girls	8	15.00	(2.83)	13.13	(5.30)	4.75	(4.23)	10.50	(6.63)
Behaviourist									
Boys	8	11.75	(4.50)	10.63	(6.87)	4.63	(5.32)	4.50	(5.88)
Girls	8	11.25	(4.74)	8.50	(5.73)	3.50	(3.07)	5.00	(6.53)
Home-based									
Boys	8	9.50	(4.72)	7.88	(5.49)	7.13	(6.88)	4.50	(6.61)
Girls	10	11.00	(5.60)	12.20	(4.92)	2.70	(1.77)	9.90	(7.31)

TABLE 6 Means and standard deviations: Preschool and non-preschool children

	N Subjects	Reproduction Mean	Reproduction SD	Transposition I Mean	Transposition I SD	Transposition II Mean	Transposition II SD	Transposition III Mean	Transposition III SD
Preschool									
Boys	26	12.00	4.35	7.31	5.86	5.39	5.79	5.42	5.72
Girls	28	13.96	3.70	12.75	4.03	4.18	3.53	8.61	6.72
Non-preschool									
Boys	13	9.39	5.62	8.69	5.87	4.77	5.02	6.54	6.57
Girls	18	11.17	5.07	10.06	5.63	2.72	3.61	7.72	6.62

However, the transpositions of the matrix required the child to recognise not only the spatial orientation of the whole pattern itself, with the key block as the indicator, but the direction of each series and class of blocks. The majority chose as their first block one of the adjacent blocks to the one remaining on the board except in the reproduction task when block 1 was selected to start 60% of the time, and block 4 (the largest pale block) 15% of the time. Table 7 shows the block preferences in the three transpositions. Size was the more salient dimension and most frequently chosen spontaneously as the basis for sorting blocks in the warm up session. However, the usefulness of selecting a particular dimension varied. When a corner block was chosen initially the child took a risk which did not always prove profitable. The choice was evidence that an overall spatial sense was being developed which might at a later stage lead to a greater degree of flexibility through the use of reference points (Healey, 1978a).

TABLE 7 Strategies: Frequencies of Preferences for Initial Blocks in Transposition Problems and the Percentages of Correct Reconstructions Following their Choice

Salience	Trans. I		Trans. II		Trans. III	
	N choice	N successful* result	N choice	N successful* result	N choice	N successful* result
Adjacent in size series	35	17	23	7	14	8
Adjacent in colour	25	17	17	1	12	3
Marker or corner block	11[2]	5	38[2]	3	18[1] 32[2]	6 12
Total	71	39	78	11	76	29

* Criterion of success
More than 12 blocks placed correctly in matrix

[1] = column marker (size)

[2] = corner marker

The majority of subjects chose to continue to work along the size series holding the colour class constant but this technique sometimes led to insufficient attention to the discrimination of colour differences so that blocks of the same size were often placed in the wrong colour order while maintaining their correct size order. Table 8 shows that this procedure was more successful than any other technique and that dependence on the size differences tended to increase and then to drop slightly in the last subtask. The problem in which colour differences were most critical was Transposition III, but even here, once the direction of the colour series had been decided, the majority of our subjects reverted to working in the size series.

c) Errors

These were non-systematic on the whole. The one exception was the mirror image error. This is a spatial error because it does not retain the rotation on a plane. It occurred in 41% of cases in Transposition II, and only 2% in Transposition I. In both of these problems the key block was in an ambiguous position on the diagonal axis of symmetry. Those children who made this sytematic error treated the problem analytically and relied on ordering of series in their solutions, but failed to observe the spatial rotation. The highest incidence of the error was made by children in the Cognitive program who tended to respond in this way to Transposition II in the majority of cases (11 children out of the sample of 35).

TABLE 8 Series Preference and Success: Percentage of Preferences for Working Along a Particular Series*

Salient series	Reproduction		Transpos. I		Transpos. II		Transpos. III	
	% choice	% success	% choice	% success	% choice	% success	% choice	% success
Size	52	33	58	30	62	9	56	28
Colour	30	17	22	15	23	1	19	5
Mixed	18	8	30	6	14	2	25	3
Total	100	58	100	51	100	13	100	36

* criterion for success 12 or more blocks correctly placed

The overall incidence of systematic error in this problem (41%) is similar to the 43% error rate of a sample of 21 children from 5 to 8 years tested at West Ryde Public School (Healey, 1978a).

Task 3

Number operations

During the preschool year we had attempted some simple number operations with concrete materials but the instructions were variously interpreted (Healey, 1978b).

The interpretation of verbal instruction is an issue which has not been sufficiently researched but it is a crucial component of the 'clinical' testing method. With this method, popularised by Piaget and his followers, the child is asked a 'why', 'what' or 'how' question which is specific to an action problem and if he cannot answer it, probes are provided which assist him to break the task down into easier components. Recording his responses at every level is of the utmost importance because this will provide the key to his level of understanding. Although Piaget did not appear to standardise the order of his probes (Piaget, 1952), it is possible to do so after a trialling of the task with a number of children. Our experiences with the preschool children showed the importance of this and so we developed a set of instructions and probes which branched off logically from different responses of the child (See record sheets, Appendix C, Figure 1).

We attempted also to prepare an exhaustive set of games using the same equipment, a number of wooden cars and trucks with varying numbers of seats (2, 4 and 6) into which could be fitted some wooden men (Figure 4). The pilot study indicated that from the children's view there were redunancies in the games. Addition and subtraction problems using these trucks were perceived as the same problems and there was insufficient variety in the vehicles to create many different problems on the basis of number alone.

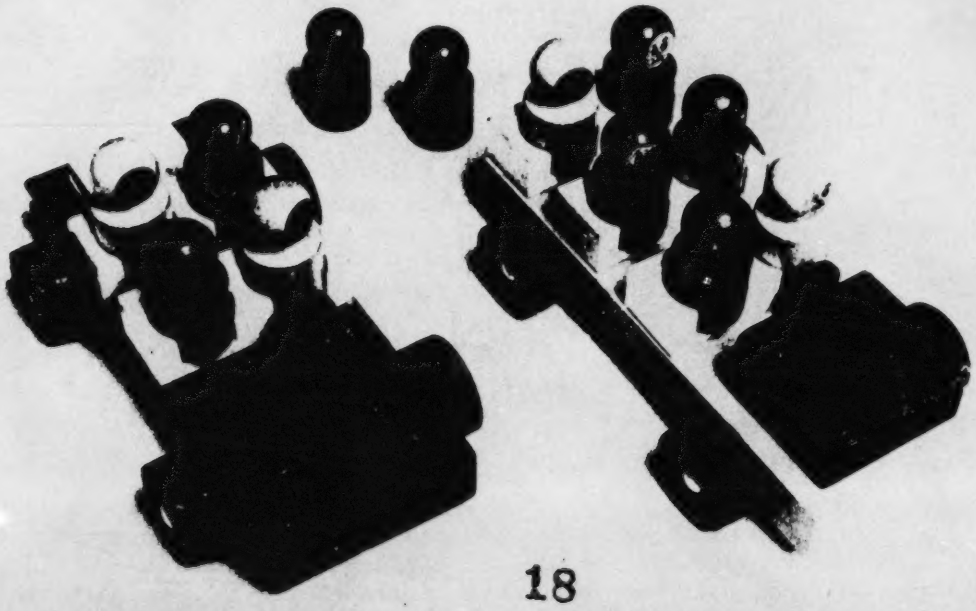


FIGURE 4 Toys Used for Number Operations Task

It seemed also that as long as the vehicles remained in view children of this age could solve many kinds of problems concretely. Consequently, instead of the anticipated range of addition, subtraction, partition, quotition and multiplication, we devised three problems with differing levels of abstraction.

a) Addition, a simple warm-up problem All of the trucks were in view (i.e., one six-seater, two four-seaters and one two-seater) with one four-seater and the two-seater filled with men (the rule was no men on the bonnet or anywhere else but in the seats). It was explained that the four-seater was empty of petrol, a fact readily appreciated during times of fuel strikes. There would not be enough petrol to send two trucks. Could we find a single truck which would hold all of the men?

This was really a warm-up since the only likely incorrect answer would be the substitution of another four-seater which might be thought to have petrol. In fact, 95% of our subjects responded correctly. The others corrected their initial response when they were reminded they could use only one truck.

b) Division

This was similar to the fire engine problem presented to the preschoolers (Healey, 1978b), but made more abstract. Also, we removed the play distractors which engrossed the five year olds but reduced their rate of progress.

We hid all of the other trucks and held up one four-seater. Twelve men were arranged at an imaginary bus stop. The question was "How many trucks like this one do we need to take all of these men to the football match?" If necessary, we assured the child that this was the only type of vehicle available for hire that day.

There were two possible correct responses - "three" would indicate correct division, but "two" might answer the hidden question "How many trucks as well as this one?" Any other answer would be wrong. The probes depended on the response. If the child said "two" the next question was "Will there be any men left?" Then "show me ".

The answers to these questions were scored and it was recorded whether the child accompanied his answer with any actions, eg. dividing the men into sets.

If the tester was in any doubt about the response there was an opportunity to allow some concrete assistance. The next probe was "How many men will fit into one truck?" This question appeared to assist some children in dividing the 12 into sets of 4. The initial question was repeated. If the child failed to find the solution he was asked "How many trips would this truck have to make?" and the child was permitted to take the appropriate action. A score was allocated to indicate the level of his response.

c) Remainder

The seats of the vehicles were structured in multiples of two which gave some support to mathematical deduction with even numbers. In spite of this, it was difficult for the children to destroy the whole configuration of the seats just as it was difficult for them to partition the set

of 12 men. This problem required the child to imagine some empty seats or some men left over. The 12 men were replaced by 10. The question was (still referring to the four-seater) "How many trucks do we need now?"

There were two possible correct answers, either "two" (i.e. with 2 men left), or "three" (with two seats empty) which might indicate the ability to break either configuration. Either answer required confirmation by explanation or action. A score was given for the verbal response and then for the concrete response. Most children needed to be asked whether there would be a remainder. Further than this they were permitted to use the demonstration truck to solve the problem over a number of trips. Those who did not take the actions spontaneously were prompted to do so. A few failed to reach a solution by the exclusion of all possibilities.

Thus there were two scores for the division and for the remainder problems: the first for a response given without use of materials and the second for the level at which use of materials provoked the response.

Results

a) The highest possible score for division was 4 and one third of our subjects attained this level of abstract response (see Table 9). Another 13% were able to reach a complete solution concretely, that is, to correspond the number of men with the number of seats by grouping and to count either groups of men or trips to obtain an answer. A few children (9%) were able to achieve the correct answer, without acting it out, after the second question. Some of these children counted the men spontaneously.

b) Thirty nine per cent of the children gave an incomplete answer to the remainder problem even after prompting - they said either "three" or "two" without reference to a remainder. Twenty one per cent recognised that there would be some seats or men left. A few children asked for a six-seater truck, recognising the difference between the four seats and the 10 men. Fifteen per cent failed to reach any solution, even with the assistance of probes.

Surprisingly, responses to the two patterns did not correlate highly with one another ($r = .29$) when only the scores for spontaneous solutions were used.

TABLE 9 Frequency of Correct Responses in Number operations - Division and Remainder (spontaneous responses only)

Score	Division	Remainder
0	37	26
1	9	6
2	7	31
3	0	17
4	27	0
Total	80	80

Task 4

Number Patterns

The aim of this set of problems was to test the ability of the child to perceive repeated patterns in number and to predict the next item in a sequence.

The materials were a set of three open-ended abacus rods. The bases of the rods could be fitted together to make a 'train' (see Figure 5).

a) One abacus was placed in front of the child and he was told "I am going to put some nuts (bolts or other acceptable word) on here and I want you to watch what I do and see if you can guess what comes next". 1, 2 and 3 nuts were placed on the first, second and third spikes. The tester asked "Do you know what I'm going to do next?" No score was given because this was a set inducing question. Another abacus was placed end to end with the first and one nut was placed on its first spike. The tester asked "Do you know what I'm going to do now?", before placing 2 and 3 nuts on the second and third spikes. When the third abacus was in position the child was asked "Do you know what I've done here?" (pointing to the spikes in turn). "Can you finish the pattern?" (pointing to the empty abacus).

b) The same procedure was adopted with 2, 4 and 6 nuts on the first three spikes and '2, blank, 6' on the next. The child had to fill in the missing number and complete the pattern on the third abacus.

FIGURE 5 Apparatus for Number Patterns

c) This time numbers written on small cards were placed next to and in front of each spike of the first abacus. They were 3, 5 and 7. The tester said "Can you make this pattern?"

d) The last pattern involved multiplication. The tester arranged nuts on the first abacus in the order 1, 2 and 4, then the same arrangement on the second abacus. She gave these to the child and said "All of these are yours. This one is mine" (pointing to the empty abacus). "Can you give me as many nuts (or other name) on here as you have on those?" (pointing to the two full arrangements). "Give me as many nuts as you have and make them look a bit like yours" (pointing to the stepped arrangement of the other nuts). The instructions were repeated "Both of those are yours. I want to have as many nuts as you have. Put them all on here".

If the child counted off the correct number but did not arrange them the tester said "Fine, but make them look like yours, see?" Incorrect responses were drawn on a prepared sheet (Appendix C, Figure 2).

Results

a) Frequency of correct repeated pattern construction

These results are set out in Table 10 as frequencies of correct, partially correct and incorrect responses. A partially correct response was one in which the nuts on one or more of the spikes were the correct number. In the case of the numerals, some children responded to them by placing the correct numbers opposite the cards but failing to perceive that the pattern should be completed over the remaining three spikes in the same way.

The wrong responses included failure to respond. The score for arrangement in the multiplication task was given when the child preserved any stepped arrangement even if the numbers were not in proportion. Some of the children who were mathematically correct did not preserve the arrangement and they were given a score of 1 also.

Responses to problems a and b correlated moderately (r = .6) and there was a lesser correlation (r = .4) between the arrangement and the correct number of the nuts placed by the child in response to problem d.

TABLE 10 Frequencies of Individual Responses to Instruction in Number Pattern Problems (N = 80)

Level of Response	a) Simple	b) Doubling	c) Numerals	d) Multiplication arrangement	d) Multiplication number
Correct	60	44	51	60	11
Partial	3	28	21	0	32*
Wrong	17	14	14	26	43

* Multiplied correctly on 1 or 2 of the 3 spikes

Task 5

Conservation of number

The nuts from the number pattern apparatus were used for a number conservation task. Two rows of ten nuts were constructed by the tester in cooperation with the child (one to one correspondence). After the child had agreed that there were the same number in each row ("Are there as many in that row as this one? Have you got more, or have I got more? Have we both got the same?"), one row was displaced a) by extending it so that the space between each of the nuts was greater, b) by bunching the alternate row.

In each case the child was asked the same set of questions and between the two transformations the rows were equalised as before.

Correct and incorrect responses were recorded and the reasons for the child's judgement were requested in each case.

Results

About 60% (compared with 30% in the preschool year) gave correct conservation judgements to either question but they were not necessarily the same children. Forty two per cent were consistent in giving correct judgements and reasons, 15% were consistent in correct judgement but not necessarily consistent in explanation and 11% judged one transformation correctly but not the other. One third of our subjects were completely wrong in their conservation judgements.

These data were grouped and analysed by Multivariate Analysis of Variance. A significant difference in response to these number problems was revealed by this analysis. There was a significant difference between preschoolers and non-preschoolers in number operations and number patterns ($p<.006$). The univariate result showed that there were differences in the remainder problem ($p<.002$), number pattern (a) ($p<.016$) and number pattern (b) ($p<.009$). In each case the group scores of the preschoolers were higher but most of the difference was in the girls' results since non-preschool girls tended to have the lowest mean scores throughout.

When the children were required to translate the written numbers into patterns the non-preschoolers were not inferior.

Preschool boys made more correct conservation judgements than any other group but the differences were not significant.

Task 6

Measurement

a) Tower building The first problem was taken from Nelson and Kirkpatrick (1975). It required only 12 identical cubes (approx. 3cm) of the same colour and a strip of cardboard to provide a perceptual anchor. The blocks were arranged in towers of 5, 1, 2 and 4 at equal intervals along the cardboard strip. The latter gave some room for manipulation but was also intended to constrain the response. However, no marks were provided on the cardboard for the positioning of the towers (see Figure 5).

The child was invited to look at the towers and notice that they were different from one another. He was asked could he make them all the same. He was then told that what we wanted was to have as many towers as there were now, but all the same size. He was invited to make them "just as big as each other".

Some children had difficulty doing this without destroying the number of towers and when this occurred the original arrangement was rebuilt by the tester and the next attempt was recorded as trial 2. The child could have as many trials as he wished but the game was stopped if he became frustrated. The number of trials and the number of block moves up to 20 were recorded and so was the final arrangement.

b) Comparison of lengths with measuring rod A card 40cm by 20cm in red was used to display three pairs of yellow cardboard strips. The members of each pair were equal in length but the pairs differed from one another (12cm, 10cm, 9cm). A Dienes rod (10cm and grooved at 1cm intervals) was placed on the table but attention was not drawn to it (see Figure 6).

The tester pointed to the yellow strips calling them sticks. They were glued on in the positions shown in Figure 6. The tester asked "Which of these sticks is the longest?" and then "Which is the shortest?" Although she recorded whether the child recognised the equivalence of each pair only the correct estimation of 'longest' was scored. A score was also given if the child used the measuring stick or part of the body spontaneously to check the lengths. Finally the child was prompted by the questions "How do you know?" and "Could you use this to find out?" A score was allocated for the style in which the measuring stick was used (eg. placing it at one end of the strip and noting the discrepancy, whether each strip was measured or only the one selected, etc.).

Results

a) About half of our subjects needed only one set of instructions to undertake the equalising of the towers (1 trial), 28% needed to have the original structures rebuilt once and 22% either twice or three times. Some children took every tower apart so that there were 12 single ones. They could not recapture the configuration required. Others had difficulty taking blocks off. They wished to build the existing towers higher and so made two equal towers of six blocks. Some compromised and made three towers of four blocks each. Others completely failed to equalise them.

19% completed the task in the minimum number of block moves (3);
14% required only one extra move to reach the correct result;
60% completed in 8 moves or less)
20% exceeded 20 block moves) not necessarily correct
66% completed the equalising task successfully on both counts, number of towers and number of blocks in each tower.

Table 11 provides a comparison of group performances according to program and sex of the child.

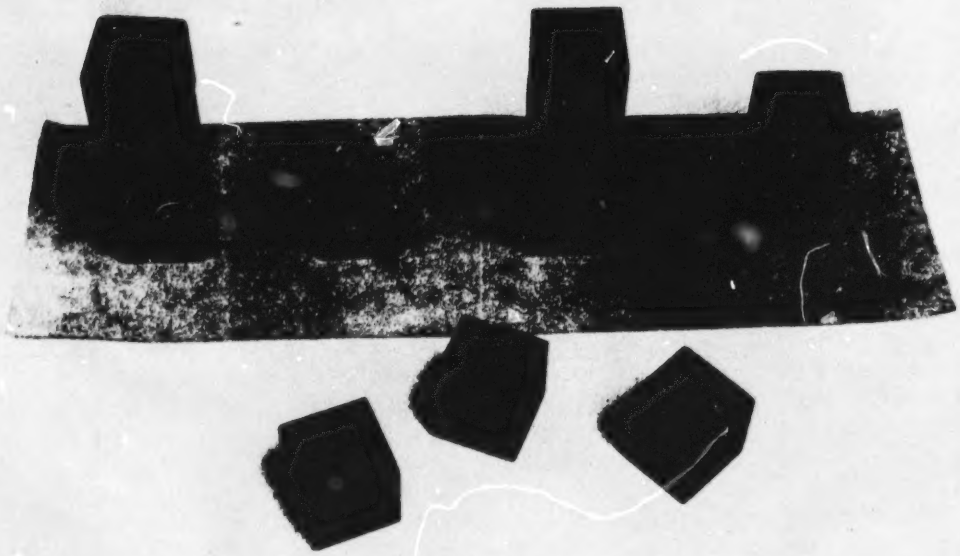


FIGURE 5 Tower building blocks task

TABLE 11 Means and standard deviations - Tower building

Program	N Subjects	Tower Trials range 1-4 $\bar{X}$	SD	Tower Moves range 3-20 $\bar{X}$	SD	Tower Result (Correct score) 1 or 2 $\bar{X}$	SD
Cognitive							
Boys	8	1.75	(1.28)	8.38	(6.82)	1.50	(0.76)
Girls	9	1.33	(1.00)	6.22	(5.47)	1.89	(0.33)
Competency							
Boys	7	1.57	(0.79)	5.29	(3.15)	1.43	(0.98)
Girls	9	1.67	(1.00)	7.44	(5.43)	1.56	(0.73)
Contemporary							
Boys	8	1.75	(0.71)	11.50	(7.25)	1.88	(0.35)
Girls	7	1.86	(0.90)	12.57	(7.83)	2.00	(0.00)
Behaviourist							
Boys	8	2.38	(1.06)	13.00	(6.41)	0.50	(0.76)
Girls	8	2.75	(1.17)	12.50	(6.55)	0.38	(0.74)
Home-based							
Boys	7	1.27	(0.76)	6.86	(6.31)	1.71	(0.76)
Girls	9	1.67	(0.71)	7.11	(5.28)	1.67	(0.71)

b) Nearly every child estimated the longest strip correctly (94%) but only about half used the measuring stick accurately in response to the question "How could you be sure?" Only two children used part of the body to check. One quarter of the children responded to some prompting in using the measuring stick but 19% failed to confirm their estimation in any way.

There was a significant difference among programs in the measurement responses (multivariate analysis of variance $p<.001$). The differences were significant for each of the tower variables, number of trials ($p<.013$), number of moves ($p<.007$), and number of correct responses ($p<.001$). Inspection of the mean scores site by site indicated a low mean correct score for the children of the Behaviourist Program in comparison with the others, and a greater mean number of both trials and moves. The only other mean score which was equivalent to the Behaviourist Program was the number of moves of individual blocks made by children of the Contemporary Australian Program. However, these children more often reached a correct end result. There were no significant differences between preschoolers and non-preschoolers in the measurement problems. Non-preschoolers tended to be more spontaneous in measuring but not in exhaustive use of the measuring stick. preschoolers tended to be more successful in equalising the four block towers.

The least successful group all told was the group of girls who had not attended preschool (N = 15).

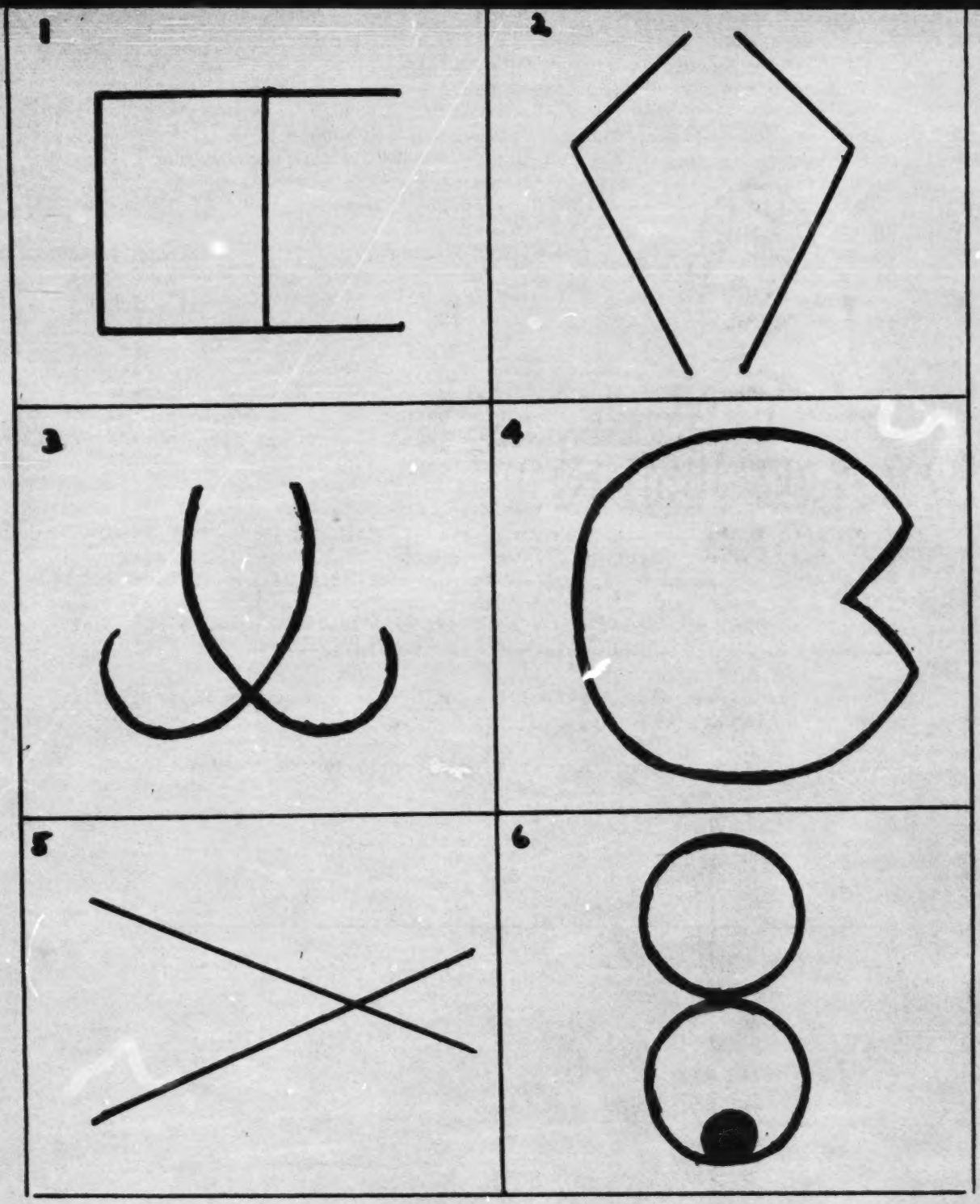
Task 7

Symmetry

Although two of the problems already discussed had included, indeed emphasised, spatial components (problems 1 and 2), it seemed important to provide an opportunity for solving a spatial problem which had no arithmetical components. The experiment was designed by Genkins (1975) and its object was the recognition of the axis of symmetry of a number of plane figures. Only six of the examples designed by Genkins were used (see Figure 7) and these were all bilaterally symmetrical. Three other designs were drawn on tracing paper so that they could be folded and checked during the practice session. Two were bilaterally symmetrical, the other not. During the practice the children were asked to pay attention to the fold line which divided the pattern.

The tester isolated each test item in turn by placing covers over the remaining patterns and asked the child whether the sides would match if this pattern were folded like the others. One of the practice items was shown again and the tester said (pointing to the fold line) "Could you draw the line where you would fold it, the line like this one?" The child was given a coloured felt-tipped pen or a thick pencil to draw with and the practice item was removed.

Each of the six patterns was exposed in turn with similar instructions. If the child drew closure lines he was asked "Where would you fold it?" and his action was noted.



NAME: _____

I.D.: _____

FIGURE 7 Symmetry Task Patterns

Results

The most difficult item turned out to be the first one presented. The desire to close the pattern proved stronger than any other consideration. Only 11% drew a correct axis of symmetry. Seventy five per cent joined the two arms of the pattern and then said it was the same both sides. Because some of the children indicated that a folding action was in the direction of their line so that the line could be said to be joining the two sides of the pattern rather than dividing it, they were credited with correct action. It was decided that all responses which connected the two outer sides of the pattern (in terms of its fold along the axis of symmetry) would be scored at a level of 1 while correct drawing of the axis would be given a score of 2. This would allow an objective judgement rather than an attempt to interpret intentions.

The six test patterns are shown in Figure 7 in the order presented to the child. Table 12 indicates the frequency of correct responses (line/contact) and the percentage of action responses of the whole group. Incorrect responses were of the kind which added detail to the pattern, closed part of it or were non-relevant to the objective.

The easiest patterns for the children to divide were 2 and 6, both of which were symmetrical around the vertical axis. This finding is of particular interest because of the research which shows that left/right discrimination emerges later than far/near discrimination (Nigl & Fishbein, 1974). The symmetry of left and right experiences in both visual and action perception has been given the credit for this and for some of the confusion in rotation problems (Huttenlocher, 1967; Pufall, 1975).

Pattern 3 was also vertically symmetrical and scores were relatively high for this pattern in spite of its low value as a 'whole'.

TABLE 12 Frequency of Responses to Symmetry Problems (N = 80)

Response type	Symmetry Problems 1	2	3	4	5	6
Correct axis	9	54	44	37	30	55
Action line	62	24	28	37	43	19
Irrelevant	15	8	14	12	13	12

Order of difficulty of symmetry tasks: Hard 1, 5, 4, 3, 6, 2 Easy

Correct responses to the horizontally symmetrical patterns were fewer and 'action responses exceeded axis responses'. One of the additional factors was the open nature of Pattern 5. It encouraged closure responses which might be interpreted as action to fold. Probably there were three reasons for the poor response to Pattern 1:

1. that it was presented first;
2. that it had an open end to a horizontally symmetrical form;
3. that the 'central' vertical line appeared to divide an incomplete figure and so make division redundant.

Consequently when the child said it was the same both sides he was probably referring to the given vertical as the axis.

There were no program or sex differences in the responses but the multivariate analysis revealed an interaction between these two ($p<.013$). One of the univariate statistics (Pattern 3) showed a significant difference ($p<.04$). Differences approached significance on responses to patterns 1 and 5. All of these were to open patterns. Three of the five lowest mean scores were on these patterns and two (Patterns 1 and 3) were obtained by the boys in the Cognitive Program. Surprisingly the same boys attained the highest mean score on Pattern 5. It is difficult to account for this result and for the low mean score for the Home-Base boys to Pattern 5.

There were no differences between preschoolers and non-preschoolers in solving symmetry problems.

Discussion of Results

a) Preschool versus non-preschool

The numbers in these subgroups were not equal; 54 preschool and 32 non-preschool. There were fairly consistent trends in favour of the preschool children, but most of these did not reach the significance level of $p<.05$. The exceptions were responses to problems in number operations and number patterns ($p<.006$). The differences were most clearly seen in three of the subtasks - remainder, simple number pattern a) and number pattern b). These results were obtained on a reduced N (48 preschool and 28 non-preschool) because illness prevented some children from finishing the test.

Some of the comparisons indicate that sex difference is an interactive factor. For instance, preschool boys are no better than non-preschool boys in the reconstruction of the matrix in two out of the four subtasks but preschool girls are consistently superior to non-preschool girls on these subtasks.

There are more conservers among preschool than non-preschool boys and either group of girls, but there is no difference between the two groups of girls in this "critical" task. It should be born in mind that conservation was scored as correct on the basis of correct judgement with or without explanation.

Regardless of the presence or absence of such cognitive structures it would seem that early school experience is extended within the classroom in the case of girls more often than boys.

b) Program Differences

There were few significant program differences and these tended to rest on responses to specific subtasks. The most significant program difference is found in the measurement responses, particularly the equalising of the block towers ($p < .001$). The two programs which contributed to most of the variance were the Behaviourist and the Contemporary Australian. Children in both programs took longer to reach a solution than any others but the children in the Behaviourist program were wrong in their solutions most often while the children from the Contemporary program were correct most often (Table 12). More accurate responses tended to be made by those children who had been in the preschool programs, overall. It may be that instructional methods increase or decrease the likelihood of certain strategies being used. The program which encourages individual interest and effort, the Contemporary, may be wasteful of time in new situations, but successful in outcomes. In contrast, the conformist approach which is reinforced by the Behaviourist program may limit the numbers of ways in which children visualise problems which are different from the usual.

c) Sex Differences

These have been explored within the dimensions of the other independent variables. The only significant differences on sex alone are in the matrix reconstruction tasks. Most of the "mirror image" responses were in Transposition II, the first of the two 90° rotations. These errors substantially reduced the mean scores on this task, but most of them were made by girls (24 out of a total of 35 errors in this subtask). While 11 of them were made by children in the Cognitive program, the majority of girls in the Cognitive and the Competency programs responded to this problem in this way. These responses are analytical in style and are mathematically correct but they ignore the spatial orientation of the whole pattern.

The incidence of this systematic error suggests that girls may respond better to perceptual training involved in dimensional differences, especially when the programs emphasise these attributes systematically. However, these results and the strategies used in the Symmetry problem also, suggest that the Cognitive program may be enhancing this type of awareness rather than the awareness of part-whole relations.

In the Representation and Memory problems preschoolers tended to make more orientational errors than non-preschoolers did, and the preschool girls made marginally more than the preschool boys. Analytical errors were somewhat more common among non-preschoolers than preschoolers (eg., additions and omissions of single blocks). These differences were not significant within the context of all of the possibilities but they may sound a warning against too much specificity in task attack at the expense of some of the larger dimensions of direction, orientation, distance, and overall configuration. While the development of analytical styles of problem solving is desirable it is necessary for the child to be able to relate the components of the problem to the characteristics or properties of the whole.

d) Spatial vs Symbolic Solutions

All of the problems had spatial components. Solutions could be attempted either by symbolic representation or by the use of imagery in most cases. Perhaps the least "symbolic" was the Symmetry task.

There were no significant differences between preschoolers and non-preschoolers in the symmetry problems and the mean scores do not show a trend in favour of either sex.

When the nature of the problem was such that counting and forming series could be incorporated into its solution, the preschoolers tended to favour these strategies. The matrix provided a set of problems which could be solved in this way, or by a combination of spatial and figural strategies. Apparently the boys more often combined them than the girls. Girls made the majority of systematic or self reference errors i.e. depended on a symbolic representation which was not adequate on its own to the solution of the transposition tasks.

3. Language and Mathematics

a) Descriptions of the Matrix

No systematic analysis was performed on the data which were derived from children's descriptions of the blocks and of the matrix design (see p) but the range of descriptors is recorded. It was noted also that more children sorted by size spontaneously than by colour tone (60% size and 49% colour). The majority of the children described the salient aspect of each class in some relevant way (83% size and 73% colour).

There was greater difficulty in finding words to match the dimensions of each series. The following lists show the limitations in range of vocabulary to describe the two series. This reflects the shortage of relevant terminology in the cultural environment for organising differences of colour tone, and also for size when a range of few comparatives is required.

colour tone	size
Simple adjectives:	
dark	big, large, fat, long,
light	little, tiny, small, skinny,
blue	short
white	medium, middle
green (light blue)	
Combined adjectives:	
light blue	tiny little
dark blue	large middle
Comparatives:	
real dark	smaller
darker	bigger
lighter	smallest
lightest	biggest
sort of white	a little bit big
creamy sort of blue	a little bit smaller
a bit dark	a little bigger
little bit dark	a bit littler
light white	the same big
lighter blue	middle sized
dark white	large middle sized
nearly dark	very big
very dark	smallest of all
	third littlest

colour tone	size
Comparatives continued	
much lighter	much smaller
half light	half little
very light	very small
Ways of describing series:	
different coloured	littler and littler
all same coloured but some are light	they go down
dark white, light white, a bit dark, dark blue	they go smaller
	go little
dark blue, a bit dark, light blue, white	little, little bit big, bigger ones, biggest
one's dark and one's darker than the other	little, middle biggest, middle, biggest
blue, light blue, green white	gets small until its really little
blue, nearly blue, light blue, white	that one's last because its smallest
white, blue, nearly dark blue, dark blue	middles to little
blue, light blue, really light blue, white	big, middle sized, a kid, a baby
	large, middle sized, little, half little
	big, littler, and littler, then little
	little and big, bigger and bigger
	big, middle sized, little and littlest
	first, second, third biggest
Bipolars on same dimension	
darker than - lighter than	from large to small
	fat/small
	that's little, that's not

Combining the two dimensions or classes

white and big
same colour biggest to smallest
same colour but all got big ones, next size, and next size
all the big ones, then the medium ones, the medium little ones, then all the little ones.

Simile and Metaphor

Only relevant simile was recorded, for example:

like stairs
like snow dropping down
the baby

There appeared to be no relationship between verbal description and result. Rather, the serial description was a challenge in its own right. Some children changed comparatives in the centre of a series, e.g., big, medium big, little, littlest, possibly because they did not have a comprehensive set of terms, but these seemed to work for them. The language is deficient in middle comparatives for a set of four. The child is also more likely to confuse the middle values of either dimension but these facts may or may not be related.

b) Language of Instruction

Hermine Sinclair-de-Zwart (1969) finds that verbal training often leads children to attend to certain aspects of a problem but does not itself lead to logical solutions. Some kinds of linguistic constructions, often semantically empty, are linked with logical behaviour. Some examples are "and, if, but, because, as much as, before" and suffixes like the comparatives "er" and "est".

However, teachers themselves do not necessarily model language which has a logical consistency or which develops with the child, acquiring greater power and flexibility. The development of tests such as these mathematical problems highlights the need for attention to the language used by the tester. The same problems which occur in devising test instructions for children occur also in the language of instruction in the classroom. The difference is that interactive, or clinical testing approaches the problem in the reverse order to most teaching. It begins with the question at a superordinate level of specificity, e.g., "Which stick is the longest?" It directs the enquiry without supplying hints about strategies. Subsequent probes suggest actions which may be taken to reach a solution, e.g., use of a measuring stick. This is a simple problem and may require only one or two prompts. The second prompt might be "What else can you do to be sure?"

i) Language and Perception

The remainder and division problems were more complex. The child needed to be able to count the seats, the men, and keep the (abstract) number of trucks in mind. The problem was designed to be an abstract one but the "given" is a concrete example of a configuration of 4 seats to help define one of the factors.

When Kindergarten children are asked how many of anything there are they generally understand that some counting must be done. What the counting must correspond to is the essence of the problem. Discrete items which can be pictured, picked up, set aside and piled up are easier to count than movements, places or spaces, parts of wholes and ordinal positions. This leads to readiness on the part of most children to count that which is easiest to count, often the objects which are central to the problem, or to use fingers as a substitute if there are no objects at hand. The language of instruction must be powerful enough to cut through the "hold" of strong perceptual features and to isolate the elements which are the focus of the problem.

The experimenter says "How many trucks like this one do we need to take all of these men to the football match?" The child, whose interest is captured by the men may answer "12". It is easy to say he has not listened to the question, and to ask him again in a way which will help focus his attention. If the second step fails, the experimenter would be wise to ask "How many men can fit in this truck?", a much simpler problem because the child can count the four seats and imagine the men left over. Many children who failed to answer the first question can now deal with it. The second communication has not only conveyed a clearer message, it has helped the child to break down the set of men.

This is not just a linguistic problem. The steps by which various children reach the solution are of interest. Mastery of a situation requires that the child can picture the division of the men and he is helped to do this by focussing techniques which are his own or someone elses. The linguistic problem lies in the segment "trucks like this one". If we require the child to conjure up the necessary configurations rather than have them supplied (and we do), we must ask either this question or the question "How many more trucks (than this)?"

However, the certainty that the child comprehends the question can only be through performance. If we ask him to take the truck, loaded, on a number of trips, he will most probably find the answer because the problem is now much easier. We want him to "do it in his head" if he can. The thorough researcher will follow up either of the responses "2" or "3" with a performance request "show me!" since either response will be correct according to the interpretation of the intial question. Some children will interpret "How many trucks like this?" as "How many (more or other) trucks like this (one)?" If the performance is inconsistent with the verbal response the child's reaction will be revealing.

ii) Language and Action

A different kind of language problem emerges from the symmetry tasks. The child's understanding of the instruction needs to be examined by the testers. They checked each of the responses by asking "Show me how you would fold it?" There are two different sorts of responses to this request. One sort clearly showed the children understood the question in a different way to the previous instruction. Now that their own actions were incorporated in folding rather than drawing they responded more concretely and more correctly. Simply thinking about the action but not doing it had not produced the correct solution.

However, language was not the source of all incorrect responses. Indeed far from it. The variety of responses to these problems showed quite clearly that a child could respond to the same instruction in three different ways. This seemed to indicate difficulty in perceiving how some figures were capable of being halved.

In conclusion

The administration of these cognitive tasks to Kindergarten children in the Mt Druitt Early Childhood Project was an attempt to obtain information about the possible effects of educational intervention upon the development of their mathematical thinking.

This type of evaluation can make a wider impact upon curriculum development and upon teaching method. The choice of instruments to determine levels of mathematical thinking is a challenge - one of applying theory to practice in schools. The problems themselves are generally appealing to children and they provide opportunities for observing and assessing perceptual development, numerical understanding and the application of logical thinking.

The analysis of the data is more difficult. Just as the classroom environments in which children learn are complex and defy the control of crucial variables for research, so also do mathematical games involve a variety of processes. A developmental researcher would do well to isolate one of the problems and test out a range of approaches. For example the addition and remainder problems could incorporate controlled variations in instruction for different groups of children. The relation between language and response could be examined in this way.

The purpose of this study was not to add to the body of developmental theory but to assess how differently our experimental and non-experimental children would respond to problems outside the normal range of classroom teaching while embodying central concepts. The differences were not as great as we might have expected.

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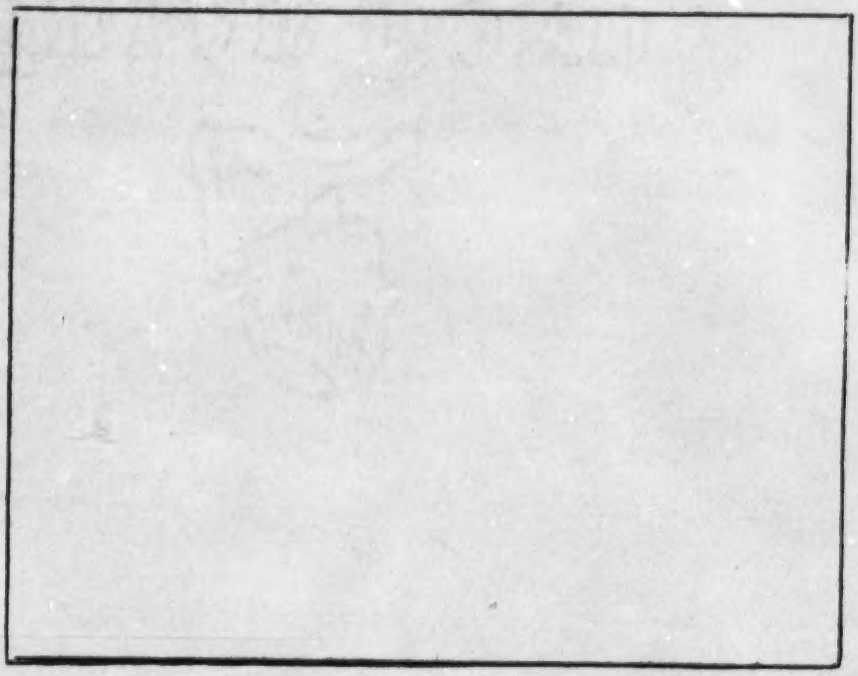
APPENDIX A
Representation and Memory

No.: _____



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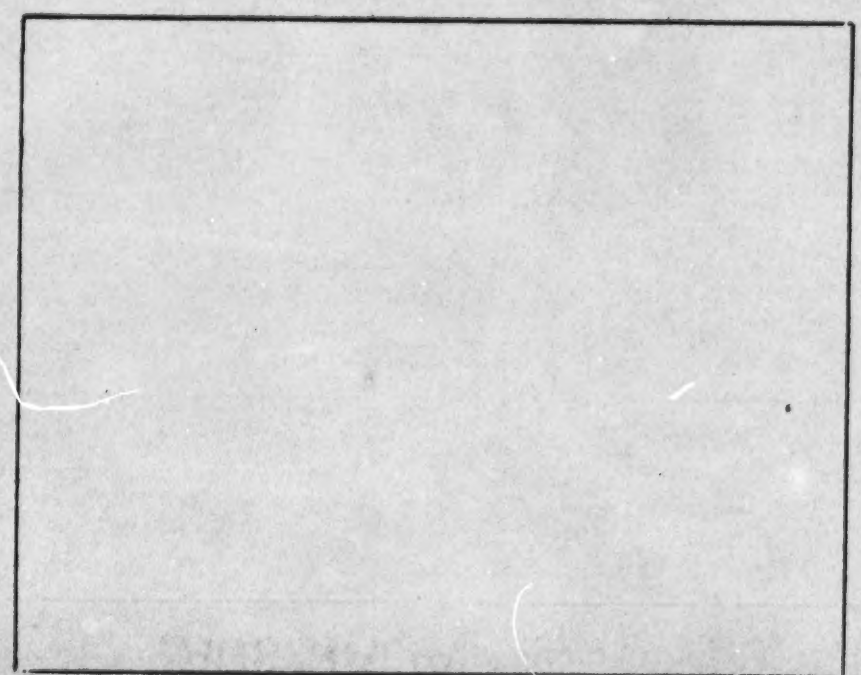

Delayed reproduction

Comments:

Yawkey, T. D. Effects of social relationships curricula and sex differences on reading and imaginativeness in young children. Alberta Journal of Educational Research, 1980, 26, 159-168.

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APPENDIX B
Scores Sheets for Matrix

Figure 1

The Matrix

Class Dimensions

a) Size

b) Colour

c) Class combination on whole pattern

Descriptions

Col. 1

Col. 2

Col. 3

Col. 4

Row 1

Row 2

Row 3

Row 4

Simple Replacement

Comments:

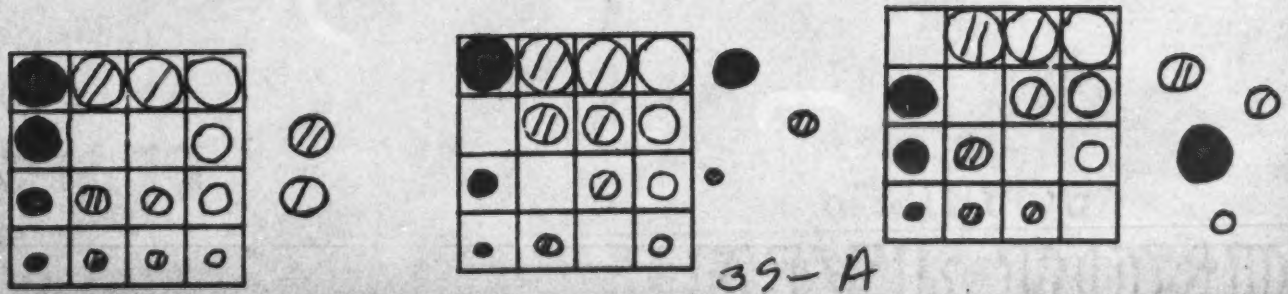
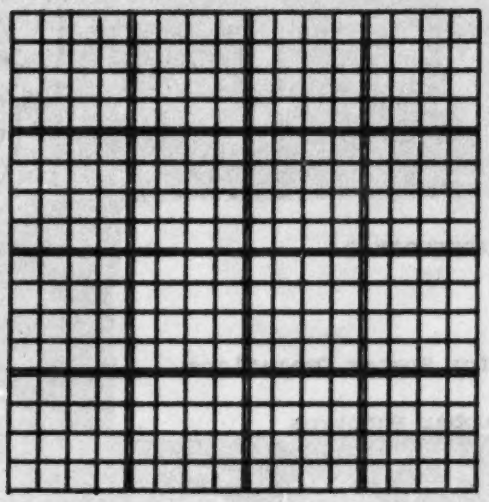
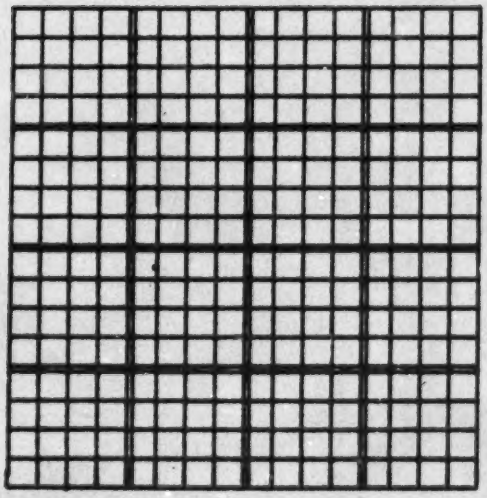


Figure 2



TIME NEEDED _____

SPONTANEOUS COMMENT:



TIME NEEDED _____

SPONTANEOUS COMMENT:

Figure 1

APPENDIX C

Score Sheets for Number Operations and Number Patterns

Name: _____

NUMBER OPERATIONS A

A. ADDITION: 4 + 2 = 6

Q. Can you find another truck which can take all of these men?

Correct Selects 4 truck

Alternative response:

B. DIVISION: $12/4 = 3$

Q. How many trucks like this one will we need to get all of these men to the football match?

Correct

Alternative response:

Q. How many men will fit in this truck?

Correct

Alternative response:

Q. How many more trucks will we need? (i.e. trucks like this)

Correct

Alternative response:

Alternative Q. How many trips would this truck have to make?

Correct

Alternative response:

C. REMAINDER: $10/4$ = 2 + 2 over

Q. How many trucks like this one?

Figure 2

Child's name: _____

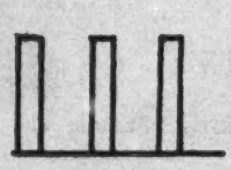
Number: _____

NUMBER OPERATIONS B

a) Number patterns

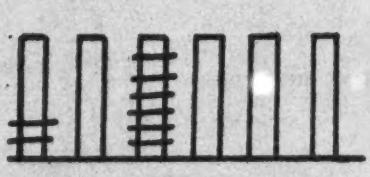
a) Copies correctly ☐

Alternative response



b) Copies correctly ☐

Alternative response



c) Copies correctly ☐

Alternative response

d) Correct ☐

Alternative response

b) Conservation

Initial equality agreed ☐

1st transformation, correct ☐

Reason _____

_____

2nd transformation ☐

Reason _____

_____

APPENDIX D

Score Sheet for Comparison of Lengths

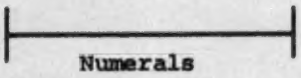
Name: _____

MEASUREMENT

I.

N of moves .....

Final arrangement



_____

II.

a) Which one is longest?

How can you be sure? - then prompt

Verbal Action

b) Which one is the shortest?

How can you be sure? - then prompt

Verbal Action

c) Which strips are the same length - as long as each other?

How do you know? - then prompt